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Thermosolutal Convection in Walters (Model B’) Elastico-Viscous Fluid in Porous Medium In the Presence of Soret and Dufour Effects

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***Abstract*: Double-diffusive stationary and oscillatory instabilities in a saturated porous layer of an elastico-viscous fluid heated and salted from below are investigated theoretically under the Darcy’s framework for a porous medium. The contributions of Soret and Dufour coefficients are taken into account in the analysis and Walters’ (model B') fluid model is used to characterize the Viscoelastic fluid. A linear stability analysis based upon normal mode technique is used to find the critical value of Rayleigh number on the onset of stationary and oscillatory convection. The effects of Soret and Dufour parameters along with other physical parameters viz., Solutal Rayleigh numbers, Lewis number and that of permeability parameter on the stability of stationary convection are studied analytically and shown graphically. For stationary convection, the analysis reveals that the Walters’ (model B') elastico-viscous fluid behaves like an ordinary Newtonian fluid. The stable solute gradient have a stabilizing effect, Lewis number and permeability parameter has a destabilizing effect on the system whereas the Soret and Dufour parameters have destabilizing and stabilizing effects on the system. The analysis also shows that the stationary convection is followed by the oscillatory convection for Walters’ (model B') elastico-viscous fluid. In the limiting cases some important results have been recovered.**

***Keywords*: Double diffusive convection, Soret and Dufour effects, Walters’ (model B') fluid, Porous medium.**

1. INTRODUCTION

A succinct account of thermal instability for single component is given in the monograph by Chandrasekhar [1] and Drazin and Reid [2]. The destabilizing buoyancy force at some critical temperature gradients makes the fluid layer unstable resulting thermal convection. The convection driven by buoyancy that is contributed by two different diffusive components, namely, temperature and solutal concentration, with differing rates of diffusion is widely known as ‘‘double diffusive convection’’ or ‘‘thermosolutal convection’’. These phenomena of combined heat and mass transfers where both temperature and solute fields contribute to the buoyancy of the fluid have many applications in the behaviour of fluids in the crust of the earth, geophysics, metallurgy, material science and petroleum engineering. Excellent reviews of the literature on double – diffusive convection in porous media and its applications can be found in Nield and Bejan [3] . Since in a double diffusive system the fluid density depends on heat and solute concentration, it leads to a competition between thermal and compositional gradients. When two transport processes take place simultaneously, they interfere with each other, producing cross-diffusion effects (Soret and Dufour effects).The flux of concentration caused by temperature gradient and the flux of heat caused by concentration gradient are known as Soret and Dufour effects, respectively. These fluxes are mainly governed by convective phenomena of the liquid phase during processing. McDougall [4] observed that the spatiotemporal properties of convection in binary mixture show quite different trends from those of the double-diffusive systems without these cross diffusion effects. The contributions of Soret and Dufour coefficients are taken into account in the present analysis

The major available literature on the phenomena of double diffusive convection with or without cross diffusion effects are mainly concerned with Newtonian fluids in non porous medium but non-Newtonian fluids in porous medium have gained tremendous interest of engineers and scientist in recent past, because of their important applications in various branches of science and technology. The fluids which exhibit both viscous and viscoelastic properties are called visco-elastic fluid. The knowledge of viscoelastic fluids flow in porous media is useful in the recovery of crude oil efficiently from the pores of reservoir rocks by displacement with immiscible water. The study of viscoelastic fluids in a porous medium has attracted the attention of large number of researchers owing to their application in petroleum drilling, the extraction of energy from geothermal regions, manufacturing of foods and paper,agriculture product storage and in many chemical engineering systems. In view of the diverse physical structures of such fluids, an extensive range of mathematical models such as Oldroyd model, Rivlin-Ericksen model, Mexwell model, Johnson-Seagalman model and the Walter-B model has been developed with different constitutive relations to simulate the hydrodynamic behavior of these non-Newtonian fluids. An eloquent exposition of viscoelastic fluid models has been presented by Joseph [5]. Oldroyd [6] proposed and studied the constitutive relations for viscoelastic fluids in an attempt to explain the rheological behavior of some non Newtonian fluids. There are many elastico-viscous fluids that cannot be characterized by Oldroyd's constitutive relations or Maxwell's constitutive relations, one such class of fluid is Walter's (model B') fluids. Walters [7] has proposed a constitutive equation for such type of elastico–viscous fluids. Walters [8] deduced the governing equation for the boundary flow for a prototype viscoelastic fluid which they have designated as liquid Bwhen this liquid has a very short memory. Walter [8] reported that the mixture of polymethyl methacrylate and pyridine at 25 0 C containing 30.5 grams of polymers per litre behaves very nearly as the Walters'(modelB) viscoelastic fluid. A porous medium is a solid with holes in it, and is characterized by the manner in which the holes are imbedded, how they are interconnected and the description of their location, shape and interconnection. However, the flow of a fluid through a homogeneous and isotropic porous medium is governed by Darcy's law which states that the usual viscous term in the equations of motion of Walters' (modelB) fluid is replaced by the resistance term

$\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\vec{q}\right]$ Where □□and □□□□are the viscosity and the viscoelasticity of the Walters' (model B□□) fluid, k_1 is the medium permeability and $\vec{q}$ is the Darcian (filter) velocity of the fluid.

Literature is replete with the various related convection problems considering variety of viscoelastic fluids (models). Sharma and Bhardwaj [9] have studies the problem of thermosolutal instability of an Oldroydian viscoelastic fluid in porous medium. Kumar et al. [10] studied the Rivlin-Ericksen elastico-viscous fluid by considering the effect of rotation and magnetic field. Wang and Tan [11] studied the stability analysis of a Soret-driven double-diffusive convection of Maxwell fluid in a porous medium using linear and non-linear stability analysis. The related thermal and thermosolutal instability problems in Walters' (Model B') elastico-viscous fluid in a porous medium are studied by Sharma and Aggarwal [12], Rana and Sharma [13], Rana and Kumar [14], Gupta and Aggarwal [15]. Shivkumara et al. [16] studied the effect of thermal modulation on the onset of convection in Walters' (Model B') viscoelastic fluid in a porous medium. Recently, Dhiman and Goyal [17] studied the stability of Soret driven double-diffusive convection problem for the case of rigid, impervious and thermally perfectly conducting boundary conditions using variational principle. This class of fluids is used in the manufacture of parts of space cafts, aeroplane, tyres, beltconveyors, rops, cushions, foams, plastics, engineering equipments and has wide applications in paper and pulp technology, petroleum engineering, geophysics, soil sciences and chemical engineering.

The problem of onset of convective motion in a double diffusive system of viscoelastic fluid particularly Walter'(model B') fluid in porous medium in the presence of Soret and Dufour effects has received very scant attention in the literature. For a saturated porous media, the phenomenon of cross diffusion is further complicated due to interaction between fluid and porous matrix and non availability of exact values of these coefficients. In most of the studies that related to the problem referred above, it has been noticed that either the inuflence of Dufour or both Soret and Dufour effect are neglected on the basis first that they are of smaller order of magnitude in liquid mixtures (Mojtabi and Charrier-Mojtabi [18],Schechter et al [19]). The cross diffusion effects, however small they may be, are present in double diffusive convections and are equally important and they have a large influence on hydrodynamic stability relative to their contributions to the buoyancy of the fluid, hence cannot be easily discarded.

keeping in mind the importance of cross diffusion effect and the growing importance of Walter'(model B') fluids, the aim of present work is to study the double- diffusive convection in a porous layer of Walter'(model B')elastic-viscous fluids in the presence of Soret and Dufour effects using linear stability analysis . The effects of Soret and Dufour parameters along with other various physical parameters viz., Solutal Rayleigh numbers, Lewis number and permeability on the stability of stationary convection are studied analytically and shown graphically.

2. FORMULATION OF THE PROBLEM AND MATHEMATICAL ANALYSIS

Let $T_{ij}, \tau_{ij}, e_{ij}, \mu, \mu', p, \delta_{ij}, q_i, x_i, d/dt$ denote respectively, the total stress tensor, the shear stress tensor, the rate-of-strain tensor, the viscosity, the viscoelasticity, the isotropic pressure, the Kronecker delta, the velocity vector, the position vector and the convective derivative. Then the constitutive relation, which is proposed and studied by Walters' [8] describing the Walters' (model B') elastico-viscous fluid is

$$T_{ij} = -p\delta_{ij} + \tau_{ij}, \qquad \tau_{ij} = 2\left(\mu + \mu'\frac{d}{dt}\right)e_{ij}, \qquad e_{ij} = \frac{1}{2}\left(\frac{\partial q_j}{\partial x_i} + \frac{\partial q_i}{\partial x_j}\right) \tag{1}$$

Here we consider an infinite, horizontal, incompressible Walter'(model B') elastico-viscous fluid saturated porous layer, of thickness d confined between two parallel horizontal planes $z = 0$ and $z = d$ which are respectively maintained at uniform temperature T_0 and T_1 $(T_0 > T_1)$ and at uniform concentrations C_0 and C_1 $(C_0 > C_1)$.The layer of fluid mixture is heated and salted from below in the the force field of gravity $\vec{g}(0,0,-g)$. A uniform adverse temperature gradient $\beta(= dT/dz)$and a uniform concentration gradient $\beta'(= dC/dz)$are maintained. The fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and the permeability k_1. The Oberbeck–Boussinesq approximation [20] is assumed to be hold, which states that the variation in density is negligible everywhere except in its association with the external force.

Utilizing the constitutive relation (1),which is proposed and studied by Walters' [8] and following De Groot and Mazur [21], McDougall [4], Sharma et.al [9], the basic equations that govern the problem under consideration are expressed as

$$\nabla.\vec{q} = 0 \tag{2}$$

$$\frac{1}{\varepsilon}\left(\frac{\partial q}{\partial t} + \frac{1}{\varepsilon}(\vec{q}.\nabla)q\right) = -\frac{1}{\rho_0}\nabla p + g\left(1 + \frac{\delta\rho}{\rho_0}\right) - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)q \tag{3}$$

$$E\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa\nabla^2 T + D_{12}\nabla^2 C \tag{4}$$

$$E'\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C = \kappa'\nabla^2 C + D_{21}\nabla^2 T \tag{5}$$

$$\rho = \rho_0\left[1 - \alpha(T - T_0) + \alpha'(C - C_0)\right] \tag{6}$$

Where $u_i = (u, v, w)$ is the Darcy velocity ; p is the pressure, μ is the coefficient of viscosity and $\upsilon = \mu/\rho_0$ is the coefficient of kinematic viscosity; g is gravity; D_{12} is the Dufour coefficient; D_{21} is the Soret coefficient; T is the temperature, C concentration, κ is the thermal conductivity and κ' is the solutal diffusivity, E is thermal capacity ratio; $E = (\rho c_p)_m/(\rho c_p)_f$,where $(\rho c_p)_f$ is the volumetric heat capacity of the fluid; E' is constant analogous to E but corresponding to solute rather than heat and $(\rho c_p)_m = \varepsilon(\rho c_p)_f + (1-\varepsilon)(\rho c_p)_s$ is the volumetric heat capacity of the saturated medium as a whole, with the subscripts *f*, *s* and *m* denoting the properties of the fluid, solid, and porous matrix, respectively; α and α' are respectively denote the thermal and concentration expansion coefficient, ρ is the density.

2.1 Basic State and its Solutions:

The initial stationary state of the system is taken to be a quiescent layer whose stability we want to examine is characterized by,

$$\vec{q} = (0,0,0), C = C_b(z), \quad p = p_b(z), \quad \rho = \rho_b(z) \tag{7}$$

Using equation (7), equations (2) to (6) yield the following stationary solution

$$\vec{q} = (0,0,0)$$

$$T_b = T_0 - \beta z, C_b = C_0 - \beta' z, \rho_b = \rho_0[1 - \alpha(T - T_0) + \alpha'(C - C_0)] = \rho_0[1 + \alpha\beta z - \alpha'\beta' z]$$

$$p_b = p_0 - \rho_0 g\left[z + \frac{1}{2}(\alpha\beta - \alpha'\beta')z^2\right] \tag{8}$$

where, p_0, ρ_0 are the values of p, ρ at z = 0.

2.2 The Perturbation Equations:

Let the initial state described by (7) be slightly perturbed so that perturbed state is given by

$$\vec{q} = (0 + u, 0 + v, 0 + w), \quad T = T_b + \theta$$
$$C = C_b + \phi, \quad \rho = \rho_0[1 - \alpha(T_b - T_0) + \alpha'(C_b - C_0) - \alpha\theta + \alpha'\phi]$$
$$p = p_b + \delta p \tag{9}$$

Where (u, v, w), $\delta\rho$, δp, θ, ϕ denote respectively the perturbations in velocity (0,0,0),density , pressure p, temperature T, solute concentration C. The change in density $\delta\rho$ caused by perturbation θ and ϕ in temperature and solute concentration is given by $\delta\rho = -\rho_0(\alpha\theta - \alpha'\phi)$

Then the linearized perturbation equations relavant to the problem are given by

$$\nabla.\vec{q} = 0 \tag{10}$$

$$\frac{1}{\varepsilon}\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}(\nabla \delta p) - \vec{g}(\alpha\theta - \alpha'\phi) - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)\vec{q} \tag{11}$$

$$E\frac{\partial \theta}{\partial t} - \beta w = \kappa\nabla^2\theta + D_{12}\nabla^2\phi \tag{12}$$

$$E'\frac{\partial \phi}{\partial t} - \beta' w = \kappa'\nabla^2\phi + D_{21}\nabla^2\theta \tag{13}$$

2.3 Normal Mode and Stability Analysis:

Analyze an arbitrary perturbation into a complete set of normal modes and assume that the perturbed quantities are of the form

$$[w, \theta, \phi] = [W(z), \Theta(z), \Gamma(z)]\exp[i(k_x x + k_y y) + nt] \tag{14}$$

Where $a = \sqrt{k_x^2 + k_y^2}$ is the resultant wave member of the perturbation, k_x and $k_y$are wave numbers along x and y directions respectively and *n* is the time constant (which is complex in general). Using equation (14), the linearized perturbation equations (10)–(13) within the framework of Boussinesq approximations, in the non-dimensional form becomes

$$\left(\frac{\sigma}{\varepsilon} + \frac{1 - F\sigma}{P_l}\right)(D^2 - a^2)\, W + Ra^2\Theta - R'a^2\Gamma = 0 \tag{15}$$

$$W + (D^2 - a^2 - EP_r\sigma)\Theta + D_f(D^2 - a^2)\Gamma = 0 \tag{16}$$

$$W + S_T(D^2 - a^2)\Theta + \left[\tau(D^2 - a^2) - \Phi\sigma\right]\Gamma = 0 \qquad \text{where } \Phi = E'P_r \tag{17}$$

together with the *boundary conditions*

$$W = 0 = \Theta = \Gamma = D^2W \quad at\ z = 0, \quad and\ z = 1$$
(Both boundaries are dynamically free) (18)

Where $D \equiv d/dz$ denotes the derivative operator; $P_r = \upsilon/\kappa$ is the thermal Prandtl number; $S_c = P_r/\tau$ is the Schamidt number; $P_l = k_1/d^2$ is permeability parameter; $F = \upsilon'/d^2$ is the viscoelastic parameter; $\tau = \kappa'/\kappa$ is the Lewis number; $R = g\alpha\beta d^4/\nu\kappa$ is the thermal Rayleigh number; $R' = g\alpha'\beta' d^4/\nu\kappa'$ is thesolutal Rayleigh number; $D_f = D_{12}\beta'/\kappa\beta$ is the Dufour parameter; $S_T = D_{21}\beta/\kappa\beta'$ is the Soret parameter. We have put the coordinates x,y,z in the new unit of length d and D = d/dz. Also we have used

$$a_* = kd; \quad D_* = dD; \quad \sigma_* = nd^2/\upsilon; \quad W = W_*; \quad \Theta = (\beta d^2/\kappa)\Theta_*; \quad \Gamma = (\beta' d^2/\kappa')\Gamma_*$$

In the resulting equations omitting the asterisks for simplicity in writing.The system of equations (15)-(17) together with boundary conditions (18) constitutes an eigenvalue problem for σ that govern thermosolutal convection in porous layer of Walters' (model B') viscoelastic fluid in the presence of Soret and Dufour effect for dynamically free boundaries.

We assume that the approperiate solution to W, Θ, Γ that satisfies the boundary conditions (18) and characterizing the lowest mode is

$$(W, \Theta, \Gamma) = (W_0, \Theta_0, \Gamma_0)\sin \pi z \qquad at\ z = 0, \quad and\ z = 1 \tag{19}$$

Substituting this solution in equation (15)-(17) and integrating each equation by parts within the range of z, we get the following matrix equation

$$\begin{bmatrix} A_{11}J & -a^2R & a^2R' \\ -1 & J + EP_r\sigma & D_f J \\ -1 & S_T J & \tau J + \Phi\sigma \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Gamma_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ where } J = \pi^2 + a^2, \quad A_{11} = \frac{\sigma}{\varepsilon} + \frac{1 - F\sigma}{P_l}, \tag{20}$$

This matrix equation has a non zero solution implies that determinant of the coefficient matrix is equal to zero which requires that

$$R = \frac{1}{a^2\left((\tau - D_f)J + \Phi\sigma\right)}\left[\left\{(\tau J + \Phi\sigma)(J + EP_r\sigma) - D_f S_T J^2\right\}A_{11}J + \left\{(J + EP_r\sigma) - S_T J\right\}a^2R'\right] \tag{21}$$

The growth rate $\sigma = \sigma_r + i\sigma_i$ is, in general, a complex quantity such that the given state of the system is *stable*, *neutral* or *unstable* according as; $\sigma_r < 0$, $\sigma_r = 0$ or $\sigma_r > 0$, for all wave numbers a^2. We are interested in the marginal stability analysis and in that case $\sigma_r = 0$ and it is apparent that the marginal state ($\sigma_r = 0$) occurs with two cases $\sigma_i = 0$ and $\sigma_i \neq 0$ When $\sigma_i = 0$ then the marginal state is characterized by the stationary convection and when $\sigma_i \neq 0$ then the instabilities are characterized by a marginally oscillatory mode and the instability sets in as oscillatory convection of growing amplitude, known as 'overstability'. we now discuss stationary and oscillatory modes of instability using the dispersion relation (21).

2.4 Stationary convection:

For stationary convection putting $\sigma = 0$ in the equation (21), we get

$$R = \frac{(\pi^2 + a^2)^2}{a^2 P_l}\left(\frac{\tau - D_f S_T}{\tau - D_f}\right) + \left(\frac{1 - S_T}{\tau - D_f}\right)R' \tag{22}$$

Here the viscoelasticity parameter F vanishes with $\sigma = 0$ implying that for a stationary convection, the Walters' (model B') elastico-viscous fluid behaves like an ordinary Newtonian fluid. Hence the effects of cross-diffusive parameters on the stationary convection of a viscoelastic fluid model behave like its effects on Newtonian fluid.

The critical stationary Rayleigh number R_c occurs when $\partial R / \partial a^2 = 0 \Rightarrow a^2 = \pi^2$ or $a_c = \pi$ .Thus, we have

$$R_c = 4\pi^2\left(\frac{\tau - D_f S_T}{\tau - D_f}\right)\frac{1}{P_l} + \left(\frac{1 - S_T}{\tau - D_f}\right)R' \tag{23}$$

Which is identical with the result reported by Motsa [22] (when P_l =1). In the absence of Soret and Dufour effects $D_f = S_T = 0$, the stationary Rayleigh number given by Equation (22). reduces to

$$R = \frac{(\pi^2 + a^2)^2}{a^2 P_l} + \frac{R'}{\tau} \tag{24}$$

which is the classical result (when P_l =1) for the double-diffusive convection in a Darcy porous medium (Nield and Bejan[3]).with critical value $R_c = \frac{4\pi^2}{P_l} + \frac{R'}{\tau}, \quad a_c = \pi$ (25)

Further, for single component fluid when $D_f = S_T = R' = 0$, the stationary Rayleigh numbers is given by equation (22) reduces to

$$R = \frac{(\pi^2 + a^2)^2}{a^2 P_l} \text{ with } R_c = \frac{4\pi^2}{P_l}, \quad a_c = \pi \tag{26}$$

Which coincides with the results (when P_l =1) obtained by (Horton and Rogers[23] ; Lapwood[24]; Nield [25] and Chandarasekher [1]).

2.4 Oscillatory Convection:

For oscillatory convection at the marginal state, putting $\sigma = i\sigma_i$, (where $\sigma_i \neq 0$ real and is the frequency of oscillation) in equation (22) which reduces to

$$R = \Delta_1 + i\sigma_i \Delta_2 \tag{27}$$

Since the Rayleigh number R is a physical quantity so it must be real hence the imaginary part of

equation (27) i.e $\Delta_2 = 0$ *(as* $\sigma_i \neq 0$*), which implies that*

$$J\left(\frac{1}{\varepsilon} - \frac{F}{P_l}\right)\left\{J(\tau - D_f)\left((\tau - D_f S_T)J^2 - EP_r\Phi\sigma_i^2\right) + J\Phi^2\sigma_i^2 + \tau JEP_r\Phi\sigma_i^2\right\}$$
$$+\frac{J}{P_l}\left\{\begin{matrix}(\tau - D_f)J^2\Phi + (\tau - D_f)\tau EP_r J^2 \\ -\left((\tau - D_f S_T)\Phi J^2 - EP_r\Phi^2\sigma_i^2\right)\end{matrix}\right\} + a^2\left[(\tau - D_f)EP_r - \Phi(1 - S_T)\right]R' = 0 \tag{28}$$

It gives the frequency of oscillations. As σ_i^2 the square of the frequency of the periodic convection is always real positive, therefore the condition for which the relation (28) gives real positive σ_i^2 will be the condition for oscillatory convection. Now the equation (27) on putting $\Delta_2 = 0$ reduces to $R = \Delta_1$

$$R = \frac{J}{a^2\left[(\tau - D_f)^2 J^2 + \Phi^2\sigma_i^2\right]}\left[\begin{array}{l}\frac{1}{P_l}\left\{J(\tau - D_f)\left((\tau - D_f S_T)J^2 - EP_r\Phi\sigma_i^2\right) + J\Phi^2\sigma_i^2 + \tau JEP_r\Phi\sigma_i^2\right\} \\ -\sigma_i^2\left(\frac{1}{\varepsilon} - \frac{F}{P_l}\right)\left\{\begin{array}{l}(\tau - D_f)J^2\Phi + (\tau - D_f)\tau EP_r J^2 - \\ -\left((\tau - D_f S_T)\Phi J^2 - EP_r\Phi^2\sigma_i^2\right)\end{array}\right\}\end{array}\right]$$
$$+ \frac{1}{\left[(\tau - D_f)^2 J^2 + \Phi^2\sigma_i^2\right]}\left[(\tau - D_f)(1 - S_T)J^2 + \Phi EP_r\sigma_i^2\right]R' \qquad (29)$$

which establishes the oscillatory Rayleigh number and clearly it depends on the cross-diffusive terms along with the other parameters of the fluid. Since the oscillatory Rayleigh obtained above varies simultaneously with σ_i^2 and a^2 therefore, it is not possible to find the critical value of oscillatory Rayleigh number analytically. For this, the expression for oscillatory Rayleigh number R given by equation (29) after substituting for σ_i^2 from equation(28), is minimized with respect to the wave number numerically, for the chosen values of other parameters.

3. RESULTS AND DISCUSSIONS

The onset of double diffusive convection in a porous layer of an elasticoviscous fluid (Walter-B liquid) heated and salted from below in the presence of Soret and Dufour effect ($D_f \neq 0, S_T \neq 0$) is examined analytically and graphically. To investigate the effects of Dufour parameter, Soret parameter, solute gradient, medium permeability and that of Lewis number τ (all positive) on the onset of stationary convection in the double diffusive system. we examine the behavior of $\frac{\partial R}{\partial D_f}$, $\frac{\partial R}{\partial S_T}$, $\frac{\partial R}{\partial R'}$, $\frac{\partial R}{\partial P_l}$ and $\frac{\partial R}{\partial \tau}$ analytically. From equation (23),we have

$$R = \frac{(\pi^2 + a^2)^2}{a^2 P_l}\left(\frac{\tau - D_f S_T}{\tau - D_f}\right) + \left(\frac{1 - S_T}{\tau - D_f}\right)R'$$

(i) $$\frac{\partial R}{\partial D_f} = \frac{(1 - S_T)}{(\tau - D_f)^2}\left[\frac{\tau(\pi^2 + a^2)^2}{a^2 P_l} + R'\right] > 0 \quad \text{for } \tau \neq D_f, (0 \leq S_T < 1).$$

Which yields that Dufour parameter has stabilizing effect on the onset of stationary convection double diffusive system for $\tau \neq D_f, (0 \leq S_T < 1)$.

(ii) $$\frac{\partial R}{\partial S_T} = -\frac{1}{(\tau - D_f)}\left[\frac{D_f}{P_l}\frac{(\pi^2 + a^2)^2}{a^2} + R'\right] > 0, \text{ if } \tau < D_f \quad \textit{and} \quad \frac{\partial R}{\partial S_T} < 0, \text{ if } \tau > D_f$$

Thus the Soret parameter has both stabilizing effect and destabilizing effect on the onset of stationary modes according as $\tau < D_f$ or $\tau > D_f$

(iii) $$\frac{\partial R}{\partial R'} = \frac{1 - S_T}{\tau - D_f} > 0, \text{ if } \tau > D_f \text{ and } 0 \leq S_T < 1$$

$$\frac{\partial R}{\partial R'} < 0, \text{ if } \tau < D_f \text{ and } 0 \leq S_T < 1$$

which implies that for stationary convection, the solutal Rayleigh number has stabilizing as well as destabilizing effect according as $\tau > D_f$ or $\tau < D_f$ and ($0 \le S_T < 1$), on the system.

(iv) $$\frac{\partial R}{\partial P_l} = -\frac{(\pi^2 + a^2)^2}{a^2 P_l^2}\left(\frac{\tau - D_f S_T}{\tau - D_f}\right) < 0 \quad \text{for } 0 \le S_T < 1 \text{ and } \tau > D_f$$

which yield that the pereamebility parameter has destabilizing effect on the onset of stationary convection for $0 \le S_T < 1$ and $\tau > D_f$

(v) $$\frac{\partial R}{\partial \tau} = -\frac{(1 - S_T)}{(\tau - D_f)^2}\left[D_f \frac{(\pi^2 + a^2)^2}{a^2 P_l} + R'\right] < 0 \quad \text{for } 0 \le S_T < 1 \text{ and } \tau \ne D_f$$

which implies that the Lewis number τ has destabilizing effect on the system for $0 \le S_T < 1$ and $\tau \ne D_f$

To have better insight of the physical problem, the variation of stationary or oscillatory Rayleigh number with square of wave number are evaluated numerically for some fixed typical values of governing parameters except for one of the varying parameter occuring in the problem. The convection curves for these parameters in (R-a)plane for different values of one of the parameter are shown in fig.1-8

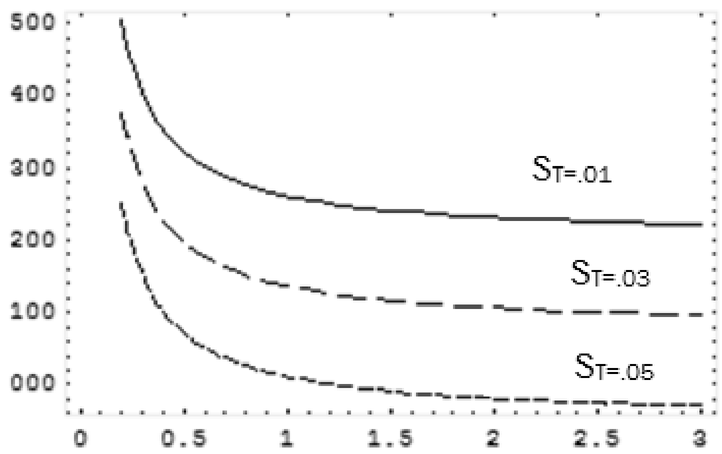

Fig 1: Variation of Stationary Rayleigh number R with wave number 'a' for fixed values of $\tau = .01, D_f = 0.002, R' = 50, P_l = 2$ for different values of Soret number S_T

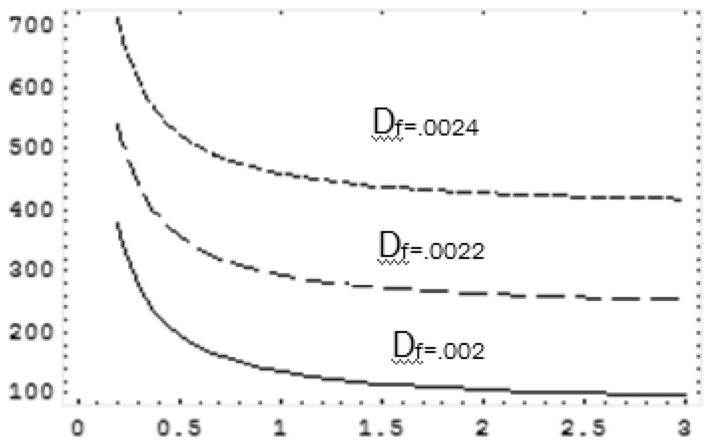

Fig 2: Variation of stationary Rayleigh number R with wave number 'a' for fixed values of $\tau = .01, S_T = 0.03, R' = 50, P_l = 2$ for different values of Dufour number D_f

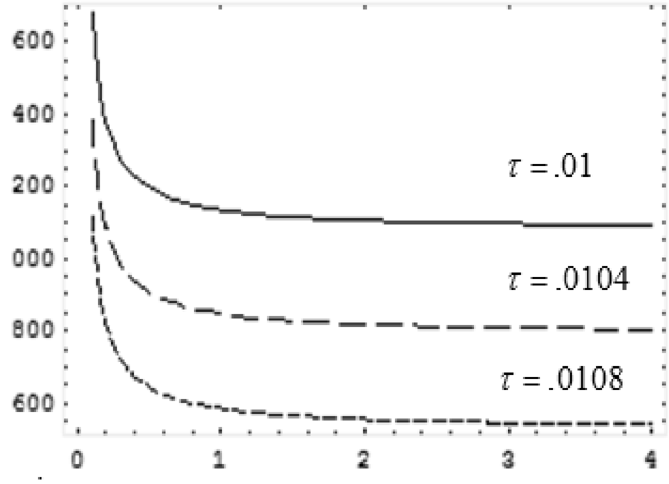

Fig 3: Variation of stationary Rayleigh number R with wave number 'a' for fixed values of $S_T = 0.03, R' = 50, P_l = 2, D_f = 0.002$ for different values of Lewis number

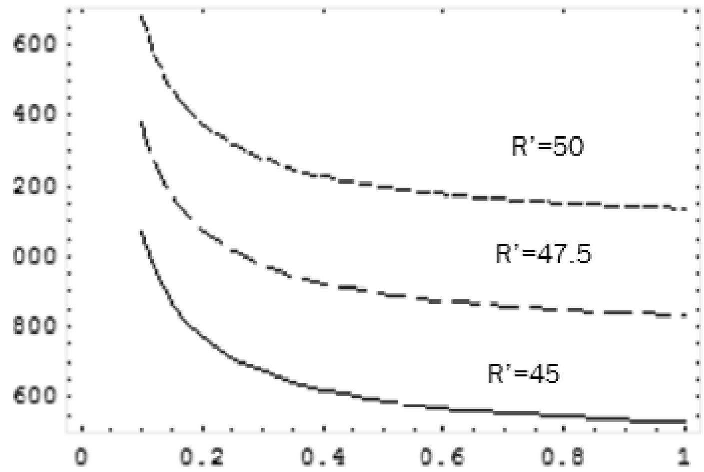

Fig 4: Variation of oscillatory Rayleigh number R with wave number 'a' for fixed values of $\tau = .01, S_T = 0.09, D_f = 0.002, P_l = 2$ for different values of solutal Rayleigh number

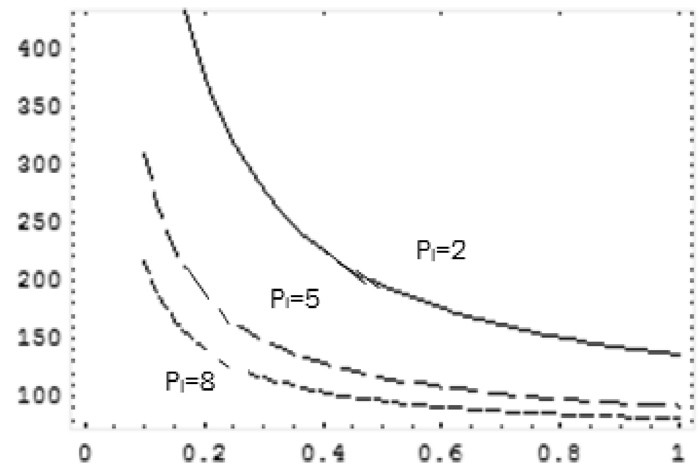

Fig 5: Variation of oscillatory Rayleigh number R with wave number 'a' for fixed values of $\tau = .01, S_T = 0.03, D_f = 0.002, R' = 50$ for different values of Permeability number

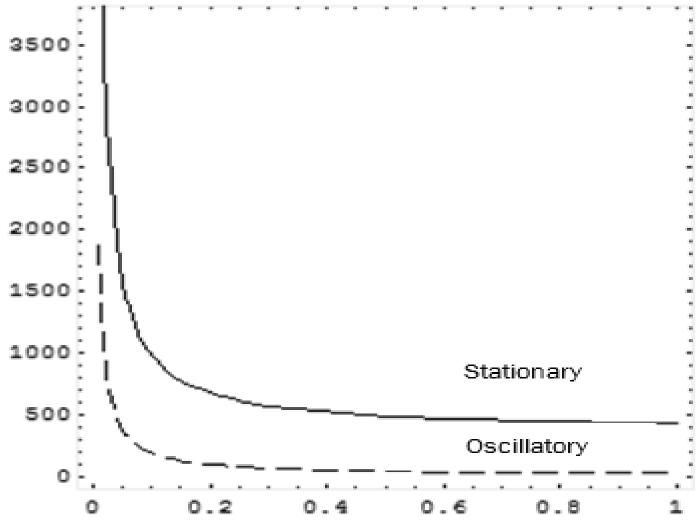

Fig 6: Variation of Stationary and oscillatory Rayleigh number R with wave number 'a' for fixed values of

$$\tau = .01, S_T = 0.03, D_f = .002, R' = 3, \sigma_i = 2, P_l = 2$$

4. CONCLUSIONS

In the present paper, the onset of double diffusive convection in a saturated porous layer of Walter (model B) elastico viscous fluid in the presence of Soret and Dufour effect is examined analytically and graphically by means of linear stability analysis. Our investigation leads to the following conclusions:

i. For the stationary convection, Walter (model B) elastic viscous fluid behaves like an ordinary Newtonian fluid due to the vanishing of the viscoelastic parameter.

ii. It has been found that in the presence of both Soret and Dufour effects($D_f \neq 0, S_T \neq 0$) The Dufour parameter has stabilizing effect whereas the Soret parameter has both stabilizing and destabilizing effect on the onset of stationary modes according as $\tau < D_f$ or $\tau > D_f$. Figures (1)-(2) support the analytical results graphically.

iii. The Lewis number τ has destabilizing effect on the onset of stationary convection whereas the solutal Rayleigh number has both stabilizing and destabilizing effect according as $\tau > D_f$ or $\tau < D_f$ Figures (3) and (4) depict these effects graphically.

iv. The permeability parameter P_l has destabilizing effect on the onset of stationary convection in the system if $0 \leq S_T < 1$ and $\tau > D_f$. Figure (5) support this effect graphically.

v. It is found numerically that the critical Rayleigh number for oscillatory convection is far less than that for the stationary convection showing that the oscillatory convection sets in much earlier than stationary convection for the chosen values of the other parameters.

Finally, it is observed that the numerical results are in close agreement with the analytical results.

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